AP Calculus
Summer Assignment
2024-25 Academic Year

Name:

Assignment Due: August 19, 2024

Google Classroom code: jtgqz52

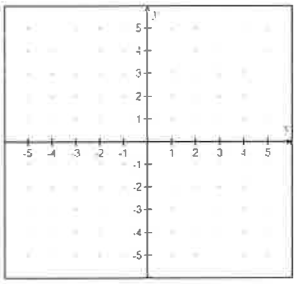
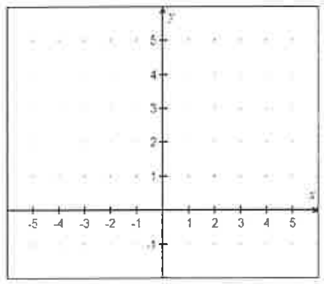
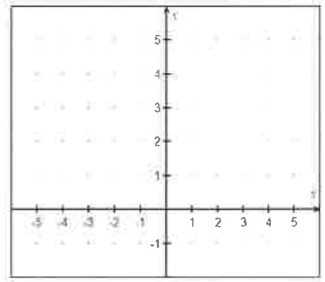
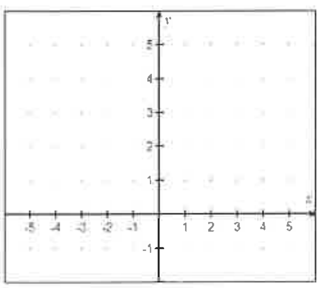
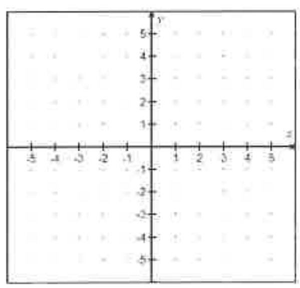
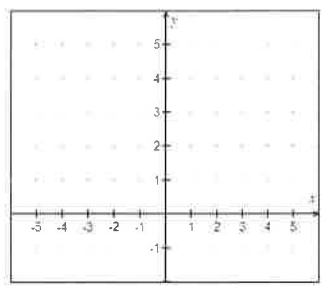
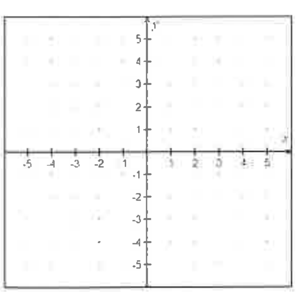
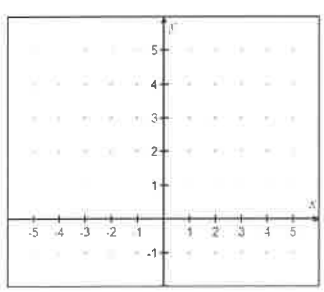
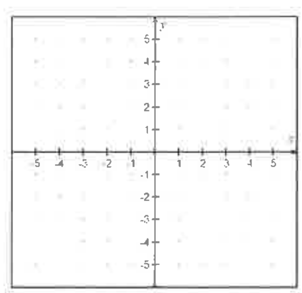
The beginning of the school year represents an opportunity to build a strong academic foundation. The object of the summer assignment is to help students achieve their maximum potential in the upcoming year. By eliminating the need to review at the beginning of the school year, classes may begin with the prescribed curriculum. Thank you for spending some time preparing for the upcoming year!

I look forward to a fun and productive year!!

Mrs Montgomery

Mrs. Montgomery

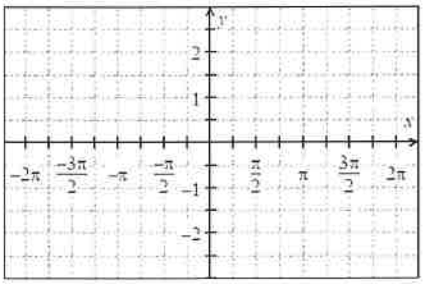
Stuff to Know Cold PreCalculus

Know Your Parent Functions		
$y = x$ 	$y = x^2$ 	$y = \|x\|$ 
$y = \sqrt{x}$ 	$y = \frac{1}{x}$ 	$y = 2^x$ 
$y = \log_2 x$ 	$y = e^x$ 	$y = \ln x$ 

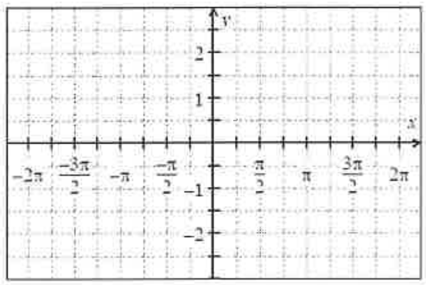
Stuff to Know Cold PreCalculus

Know Your Trigonometry Functions

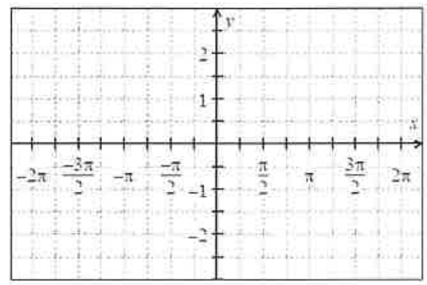
$y = \sin x$



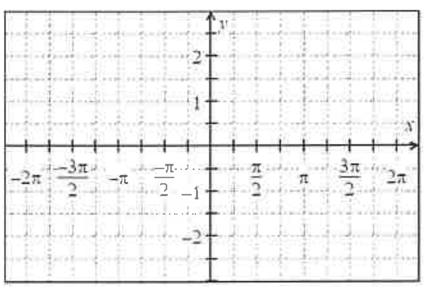
$y = \cos x$



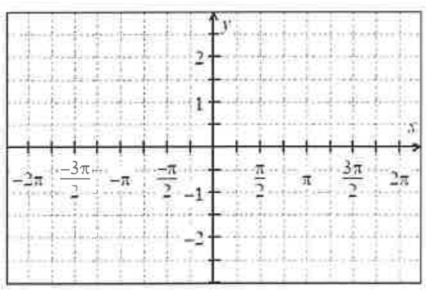
$y = \tan x$



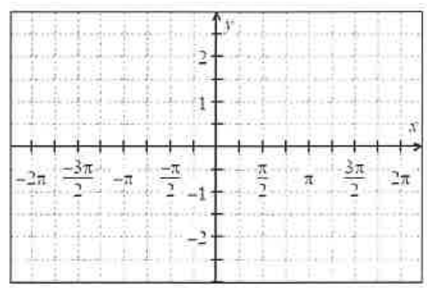
$y = \csc x$



$y = \sec x$



$y = \cot x$



θ Radians	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π
sin θ					
cos θ					
tan θ					

Where Trig Functions are Positive

QI	QII	QIII	QIV

Even Functions

If $f(x)$ is even, then $f(-x) =$

$\cos(-\theta) =$

$\sec(-\theta) =$

Odd Functions

If $f(x)$ is odd, then $f(-x) =$

$\sin(-\theta) =$

$\csc(-\theta) =$

$\tan(-\theta) =$

$\cot(-\theta) =$

Trig Identities

Reciprocals:

$csc\ x =$

$sec\ x =$

$sin\ x\ csc\ x =$

$cos\ x\ sec\ x =$

Pythagorean:

$sin^2x + cos^2x =$

$1 + tan^2x =$

$cot^2x + 1 =$

Double Angle:

$sin\ (2x) =$

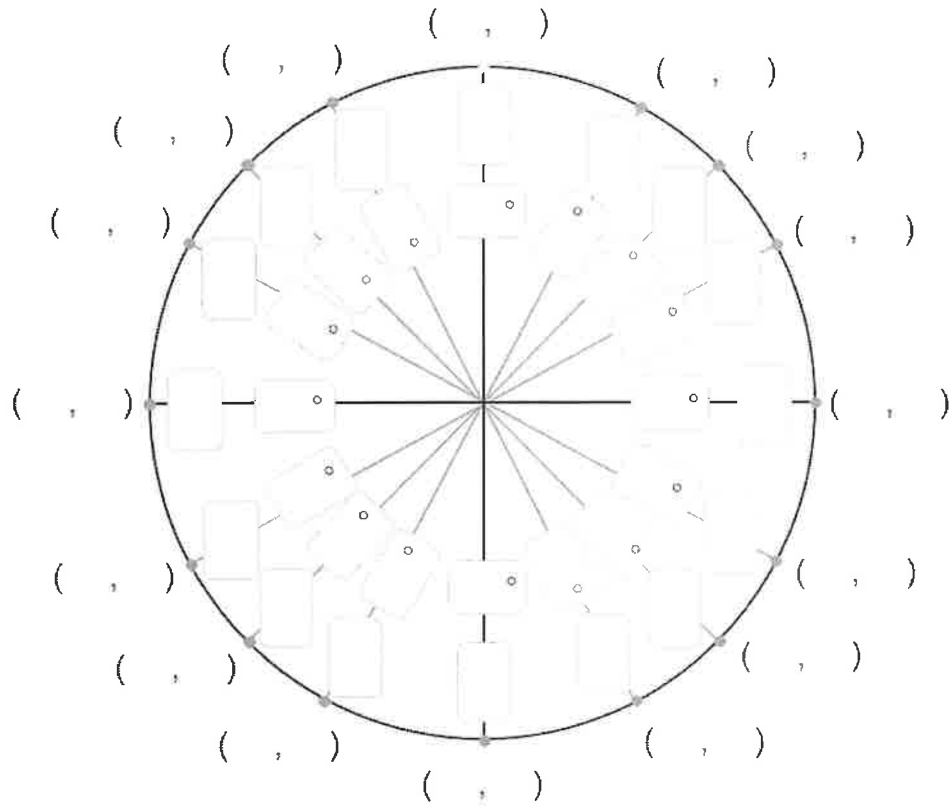
$cos(2x) =$

$=$

$=$

$\tan(2x) =$

Unit Circle



$(x, y) = (\cos\theta, \sin\theta)$

Domain Restrictions on Inverse Trig Functions

$y = \sin^{-1}(x)$	$y = \cos^{-1}(x)$	$y = \tan^{-1}(x)$

Sum Formulas	Difference Formulas
$\sin(A + B) =$	$\sin(A - B) =$
$\cos(A + B) =$	$\cos(A - B) =$
$\tan(A + B) =$	$\tan(A - B) =$

Decomposition of $N(x)/D(x)$ into Partial Fractions

$$\frac{N(x)}{D(x)} = (polynomial) + \frac{R(x)}{D(x)}$$

1. Use long division to obtain proper form when the degree of the numerator degree is greater than the degree of the denominator.
2. Factor the denominator as a product of linear factors or irreducible quadratic factors.
3. Write a partial fraction with a constant numerator for each linear factor of the denominator.
4. Clear the denominator.
5. Equate coefficients to write a linear system.
6. Solve the system to find the partial fraction decomposition.

Rules for Exponents and Radicals

$b^0 =$	$b^m b^n =$	$(ab)^n =$
$\sqrt{b}$	$\sqrt[n]{b} =$	$\sqrt[n]{b^m} =$
$b^{-n} =$	$\sqrt[n]{ab} =$	$(b^m)^n =$
$\frac{b^m}{b^n} =$	$\left(\frac{a}{b}\right)^m =$	$\sqrt[n]{\frac{a}{b}} =$

Rules for Logarithms

Change from Exponential to Log	Change from Log to Exponential
$b^x = y$ $e^x = y$	$\log_b y = x$ $ln\, y = x$
$\log_b 1 =$	$\log_b b =$
$\log_b b^n =$	$\log_b M^n =$
$\log_b(MN) =$	$\log_b\left(\frac{M}{N}\right) =$

Change of Base Formula: $\log_c a =$

Formula Sheet

Reciprocal Identities: $\csc x = \dfrac{1}{\sin x}$ $\sec x = \dfrac{1}{\cos x}$ $\cot x = \dfrac{1}{\tan x}$

Quotient Identities: $\tan x = \dfrac{\sin x}{\cos x}$ $\cot x = \dfrac{\cos x}{\sin x}$

Pythagorean Identities: $\sin^2 x + \cos^2 x = 1$ $\tan^2 x + 1 = \sec^2 x$ $1 + \cot^2 x = \csc^2 x$

Double Angle Identities: $\sin 2x = 2\sin x \cos x$

$\tan 2x = \dfrac{2\tan x}{1 - \tan^2 x}$

$$\begin{aligned}\cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1\end{aligned}$$

Logarithms: $y = \log_a x$ is equivalent to $x = a^y$

Product property: $\log_b mn = \log_b m + \log_b n$

Quotient property: $\log_b \dfrac{m}{n} = \log_b m - \log_b n$

Power property: $\log_b m^p = p \log_b m$

Property of equality: If $\log_b m = \log_b n$, then m = n

Change of base formula: $\log_a n = \dfrac{\log_b n}{\log_b a}$

Derivative of a Function: Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \dfrac{f(x+h) - f(x)}{h}$

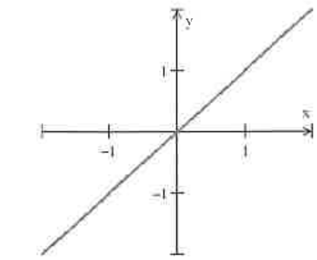
Slope-intercept form: $y = mx + b$

Point-slope form: $y - y_1 = m(x - x_1)$

Standard form: $Ax + By + C = 0$

Name: ____ Set #1: Functions

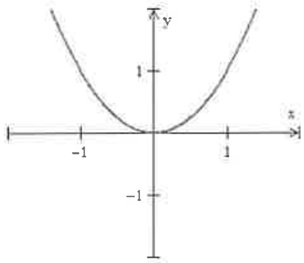
1. State the domain and range for each of the twelve basic functions:



$y = x$

D:

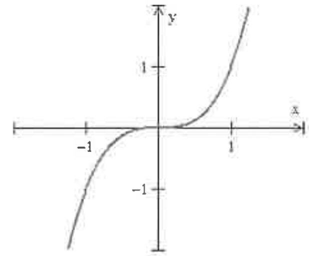
R:



$y = x^2$

D:

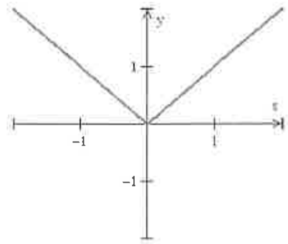
R:



$y = x^3$

D:

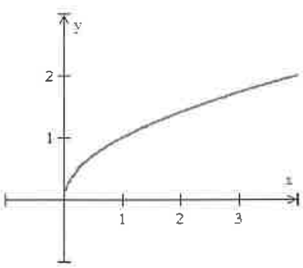
R:



$y = |x|$

D:

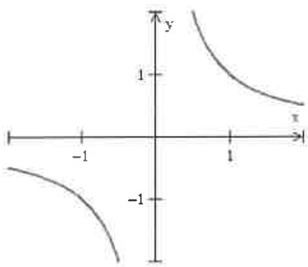
R:



$y = \sqrt{x}$

D:

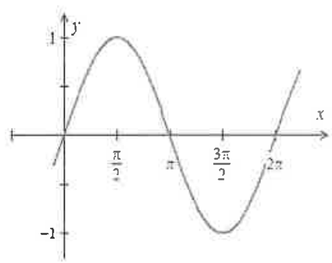
R:



$y = \frac{1}{x}$

D:

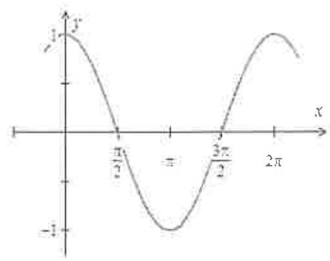
R:



$y = \sin x$

D:

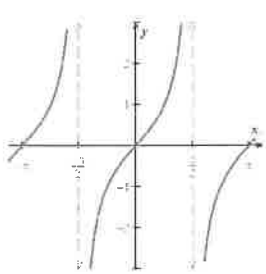
R:



$y = \cos x$

D:

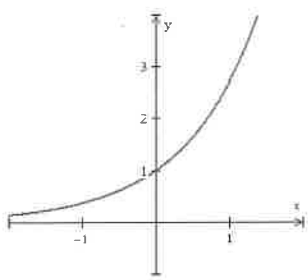
R:



$y = \tan x$

D:

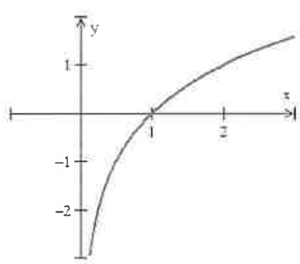
R:



$y = e^x$

D:

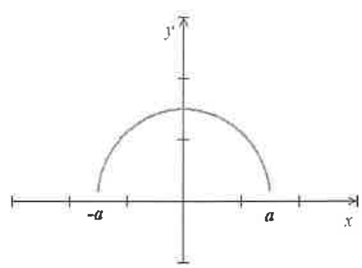
R:



$y = \ln x$

D:

R:



$y = \sqrt{a^2 - x^2}$

D:

R:

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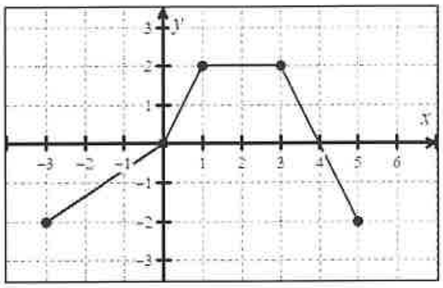
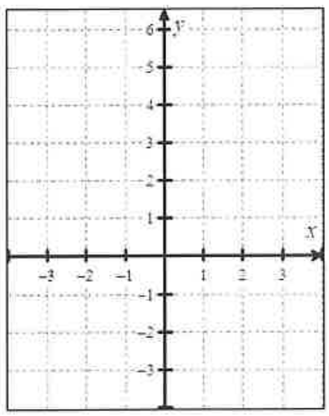
2. Which of the twelve basic functions are even?	3. Which of the twelve basic functions are odd?
4. Name the functions that have vertical asymptotes.	5. Name the functions that are bounded below.
6. Find the average rate of change of $f(x) = 3x^2 - 5$ from $x = 2$ to $x = 1$ to $x = 4$	7. Write an equation for the line that passes through the points $(8, -1)$ and $(-4, -10)$
8. If $f(x) = \sqrt{x+2}$ find the equation of $f^{-1}(x)$.	9. Does the relation represent a function? State the domain and range. $\{(-3,-5),(2,4),\ (0,2),(1,4),(3,5)\}$
10. Find the domain of the function. $f(x) = \frac{x}{x^2-4}$	11. Find the domain of the function. $g(x) = \sqrt{8-2x}$
12. If $f(x) = x^3$ describe the transformations for $g(x) = -3(x+2)^3 + 1$	13. Express the function $h(x) = \sqrt[3]{3x+2}$ as a composition of two functions f and g.

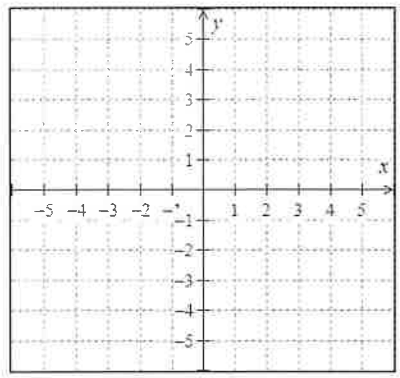
14. Given $g(x) = \begin{cases} \sqrt{x-3} \; ; x > 3 \\ 3 - x \;\; ; x \leq 3 \end{cases}$ find: a. $g(7)$ b. $g(-2)$	15. Find $f(g(x))$ given $f(x) = x^2 - 3x$ and $g(x) = 2x - 1$.
16. Given $f(x) = 3x - 2$ and $(x) = \sqrt{x-1}$; find $f + g, f - g, fg, f/g$ and state the domain for each function.	17. Use a graphing utility to determine the intervals where the function is increasing and decreasing. $f(x) = \frac{1}{3}x^3 - x^2 - 8x + 3$
18. The regular price of a laptop computer is x dollars. Let $f(x) = x - 250$ and $g(x) = 0.8x$. Describe what the functions f and g represent in terms of the price of the laptop.	19. Use the functions from question 18 to find $f(g(x))$ and $g(f(x))$. Which composite function represents the greatest discount for the laptop?

20. Use the graph of f in the figure below to draw the graph of its inverse function.

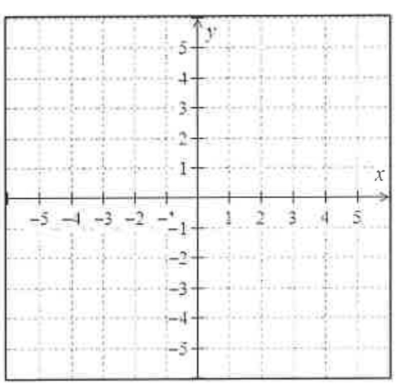




21. Sketch the piecewise function on the axes below. $g(x) = \begin{cases} \frac{x^2-9}{x-3} & if\ x \neq 3 \\ -2 & if\ x = 3 \end{cases}$



22. Find the inverse of $f(x) = x^2 - 3$, for $x \geq 0$. Sketch both functions in the grid below.



23. The function $f(x) = 0.4x^2 - 36x + 1000$ modes the number of accidents, $f(x)$, per 50 million miles driven as a function of the driver's age, x, in years, for driver's ages $16 \leq x \leq 74$. Use a graphing utility with a view window from $[0, 100][0, 1100]$.

a. For what value of x does the graph reach its minimum?

b. What is the minimum value of y? Explain what this value means in context of the problem.

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Use the Leading Coefficient Test to determine the end behavior of the graph of the polynomial function.

1. $f(x) = -4x^3 + 5x^2 - 2$	2. $g(x) = x^4 + 2x^2$

Find all the zeros of the function by factoring.

3. $f(x) = -2x^3 + 4x^2 - x + 2$	4. $g(x) = x^4 - x^2 - 12$

Use synthetic division to find all the zeros of the function.

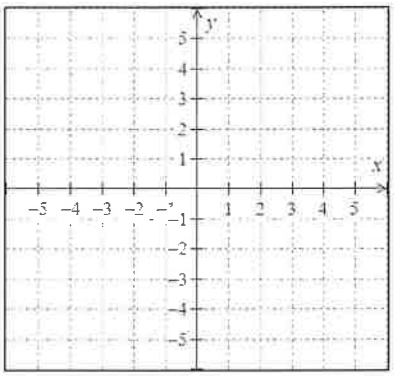
5. Given that 2 is a zero of the polynomial, find the remaining zeros for $f(x) = 2x^3 - 5x^2 + x + 2$	6. Given that $\frac{1}{3}$ is a zero of the polynomial, find the remaining zeros for $f(x) = 3x^3 - 4x^2 - 17x + 6$

7. Use the Intermediate Value Theorem to show that the graph of the function has a zero in the given interval.
$f(x) = x^3 + 3x^2 - 2x - 5; -4 \leq x \leq -3.$

8. Find all the real zeros of the function
$f(x) = x^3 - 2x^2 + 7x + 30$

9. An umbrella maker has been experimenting with a different parabolic shapes. A particular umbrella that is 14 feet wide at the bottom and is 6 feet high in the center.
   a. Find an equation of the parabola that models the curve of the umbrella. (Assume the origin is at the center of the parachute.
   b. How far from the center of the umbrella is the umbrella's surface 3 feet lower than in the middle?

10. Find the values of x at which f is not continuous. State the domain and range, sketch the function and show all asymptotes. $f(x) = \frac{2x-5}{x^2-2x+1}$



11. List the possible rational zeros of the function: $f(x) = 5x^4 - 6x^3 + x^2 - x - 3$

12. Find the intercepts for $f(x) = \frac{2x^2 - x - 3}{x^2 - 4}$

13. Find a polynomial with real coefficients that has zeros at $\{-2, 3i\}$.

14. Identify any vertical and horizontal asymptotes for the graph of the function $f(x) = \frac{x+2}{x^2 - 2x - 15}$.

15. A swimming pool is 8 feet longer than it is wide. The pool is surrounded by a walkway of width 4 ft. The combined area of the pool and the walkway is 1280 ft^2. Find the dimensions of the pool without the walkway.

16. A new strain of flu is discovered. After x days of exposure, the number of particles in our bodies, $f(x)$, in billions can be modeled by the polynomial function $f(x) = -0.75x^4 + 2x^3 + 6$. Using the Leading Coefficient Test, what does the right end behavior of the function suggest about the number of viral particles in our body over time?

Name: ____ Set #4: Trigonometric Functions

1. Find the length of the arc on a circle of radius 10 inches, intercepted by a 120° central angle. Express arc length in terms of π. Then round your answer too two decimal places.	2. The radius of a tractor wheel is 24 inches. If the wheel rotates through an angle of 300°, how many feet did the tractor move? Round your answer to the nearest foot.
In questions 3 – 4: θ is an acute angle and $\sin\theta$ and $\cos\theta$ are given. Use identities to find the remaining trigonometric functions. Rationalize the denominators, if necessary.	
3. $\sin\theta = \frac{5}{13}$, $\cos\theta = \frac{12}{13}$	4. $\sin\theta = \frac{\sqrt{5}}{3}$, $\cos\theta = \frac{2}{3}$
In questions 5 – 6 find the exact value of each of the remaining trigonometric functions of θ.	
5. $\tan\theta = -\frac{5}{12}$, $\cos\theta < 0$	6. $\csc\theta = -3$, $\tan\theta > 0$

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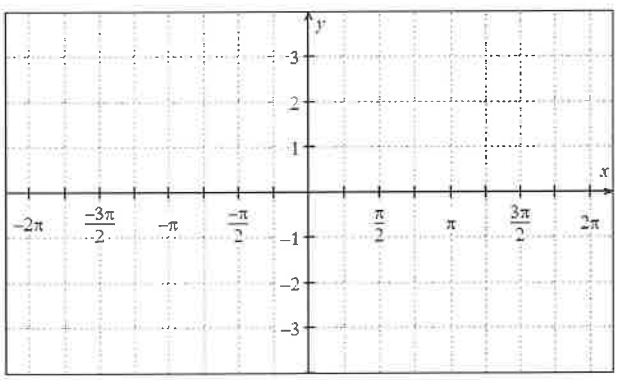
7. Complete the following.

$\sin\frac{\pi}{3}$	$\tan\frac{\pi}{2}$	$\cos\frac{3\pi}{4}$	$\sec\frac{5\pi}{6}$	$\cot\frac{5\pi}{3}$
$\tan\frac{5\pi}{4}$	$\csc\frac{2\pi}{3}$	$\cos 5\pi$	$\sin\left(-\frac{3\pi}{4}\right)$	$\sec\left(-\frac{\pi}{2}\right)$
$\cot\frac{7\pi}{6}$	$\tan\pi$	$csc\left(-\frac{5\pi}{6}\right)$	$\cos\left(-\frac{3\pi}{4}\right)$	$\tan\left(-\frac{2\pi}{3}\right)$

Graph each equation carefully. Give the period, amplitude, and asymptotes.

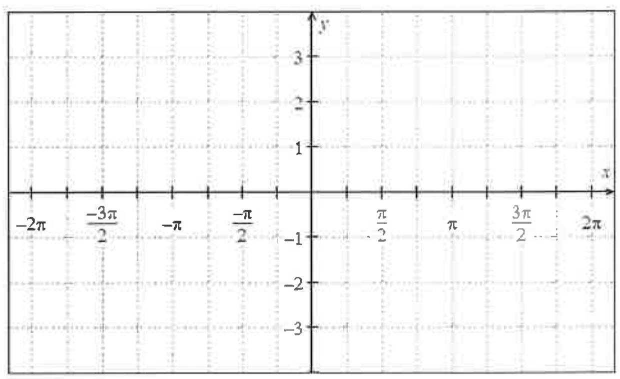
8. $y = \sin\left(x + \frac{\pi}{2}\right)$

Period: __________ Amplitude: ___________



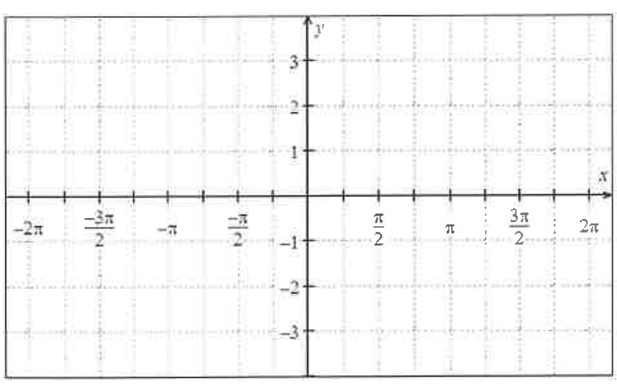
9. $y = -2\cos x + 1$

Period: __________ Amplitude: ___________



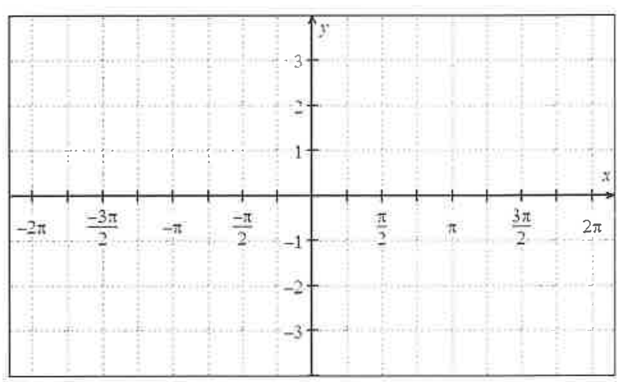
10. $y = \tan(x + \frac{\pi}{2})$

Period: __________ Asymptotes: _____________



11. $y = csc\, x$

Period: __________ Asymptotes: _____________



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Find the exact value of each expression.	
12. $\sin^{-1}\left(-\frac{1}{2}\right)$	13. $\cos^{-1}(-1)$
14. $\tan^{-1} 0$	15. $\csc^{-1}\sqrt{2}$
16. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$	17. $\sec^{-1}(-2)$
Use a calculator to find the value of each expression, in degrees, accurate to nearest degree.	
18. $\cos^{-1}\left(\frac{\sqrt{5}}{7}\right)$	19. $\sin^{-1}(2.3)$
20. $\tan^{-1}(-9)$	21. $\sec^{-1}\left(\frac{9}{4}\right)$
Use a sketch to find the exact value of each expression.	
22. $\sin\left(\cos^{-1}\frac{4}{5}\right)$	23. $\tan\left[\sin^{-1}\left(\frac{3}{5}\right)\right]$
24. The angle of depression from the top of one building to the foot of a building across the street is 53°. The angle of depression to the top of the same building is 19°. The two buildings are 80 feet apart. What is the height of the shorter building?	25. A 200 ft. guywire is attached to the top of a tower. If the wire makes a 55 degree angle with the ground, how tall is the tower?

Trigonometric Equations:

Solve each of the equations for $0 \le x < 2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0 \le x < 2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed, to rewrite the trig functions. (See formula sheet at the end of the packet.)

26. $\sin x = -\dfrac{1}{2}$

27. $2\cos x = \sqrt{3}$

28. $\cos 2x = \dfrac{1}{\sqrt{2}}$

29. $\sin^2 x = \dfrac{1}{2}$

30. $\sin 2x = -\dfrac{\sqrt{3}}{2}$

31. $2\cos^2 x - 1 - \cos x = 0$

32. $4\cos^2 x - 3 = 0$

33. $\sin^2 x + \cos 2x - \cos x = 0$